MIXING OF BOUNDARY LAYERS AT A LIQUID OR GAS INTERFACE

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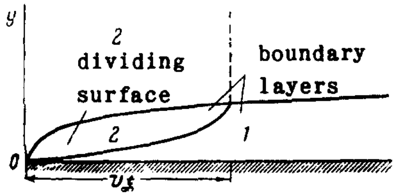
1. Consider a semi-infinite plate which at time $t = 0$ has flowing past it a stream of fluid with coefficient of viscosity μ_1 and kinematic viscosity ν_1; beginning at this moment another fluid flows past it, characterized by values μ_2 and ν_2. The speed of the fluid is assumed to be constant, equal to V_∞.



The problem consists in determining the mixing process of the boundary layers (see the figure).

In view of the character of the initial and boundary conditions, the solution for the velocity profile in the boundary layer depends upon only two parameters

$$\eta^\circ = \frac{y}{\sqrt{x \nu_1 / V_\infty}}, \qquad \xi^\circ = \frac{x}{V_\infty t}$$

Therefore by setting

$$v_x = V_\infty v^\circ{}_x (\eta^\circ, \xi^\circ), \qquad v_y = \frac{V_\infty}{\sqrt{x V_\infty \nu_1}} v^\circ{}_y (\eta^\circ, \xi^\circ)$$

we reduce the boundary layer equations to the form [2]

$$\xi^\circ \frac{\partial v^\circ{}_{xi}}{\partial \xi^\circ} - \frac{\eta^\circ}{2} \frac{\partial v^\circ{}_{xi}}{\partial \eta^\circ} + \frac{\partial v^\circ{}_{yi}}{\partial \eta^\circ} = 0 \qquad (i = 1, 2)$$

$$\xi^\circ (v^\circ_{x1} - \xi^\circ) \frac{\partial v^\circ_{x1}}{\partial \xi^\circ} + \left(v^\circ_{y1} - \frac{\eta^\circ}{2} v^\circ_{x1}\right) \frac{\partial v^\circ_{x1}}{\partial \eta^\circ} = \frac{\partial^2 v^\circ_{x1}}{\partial \eta^{\circ 2}} \tag{1.1}$$

$$\xi^\circ (v^\circ_{x2} - \xi^\circ) \frac{\partial v^\circ_{x2}}{\partial \xi^\circ} + \left(v^\circ_{y2} - \frac{\eta^\circ}{2} v^\circ_{x2}\right) \frac{\partial v^\circ_{x2}}{\partial \eta^\circ} = \nu^0 \frac{\partial^2 v^\circ_{x2}}{\partial \eta^2} \qquad \left(\nu^\circ = \frac{\nu_2}{\nu_1}\right)$$

Here v_x and v_y are respectively the velocity components along and perpendicular to the plate; indices 1 and 2 indicate the characteristics of the first and second fluid.

The boundary conditions for the system (1.1) are the following:

$$v^\circ_{x1} = 1 \quad \text{for} \quad \eta^\circ = \infty,\ \xi^\circ > 1; \quad v^\circ_{x2} = 1 \quad \text{for} \ \eta^\circ = \infty,\ \xi^\circ < 1 \tag{1.2}$$

$$v^\circ_{x1} = v^\circ_{y1} = 0 \quad \text{at} \quad \eta^\circ = 0$$

$$v^\circ_{x1} = v^\circ_{x2}, \quad v^\circ_{y1} = v^\circ_{y2}, \quad \frac{\partial v^\circ_{x1}}{\partial \eta^\circ} = \mu^\circ \frac{\partial v^\circ_{x2}}{\partial \eta^\circ} \quad \text{at} \ \ \eta^\circ = \Lambda(\xi^\circ) \left(\mu^\circ = \frac{\mu_2}{\mu_1}\right)$$

Here $\eta^\circ = \Lambda(\xi^\circ)$ is the equation of the surface dividing the fluids, which is unknown until the problem is solved; on it evidently

$$\xi^\circ (v^\circ_x - \xi^\circ) \Lambda' = v^\circ_y - \frac{1}{2} v^\circ_x \Lambda \tag{1.3}$$

Introduction of the new functions

$$v^*_{x1} = v^\circ_{x1}, \qquad v_{y1} = v^\circ_{y1} \qquad \text{for } \eta^\circ < \Lambda$$

$$v^*_{x2} = v^\circ_{x2},\ v^*_{y2} = \xi^\circ(v^\circ_{x2} - \xi^\circ)\left(1 - \frac{\rho_2}{\rho_1}\right)\Lambda' + \frac{\rho_2}{\rho_1} v^\circ_{y_2} + \frac{v^\circ_{x2}\Lambda}{2}\left(1 - \frac{\rho_2}{\rho_1}\right) \ \text{for } \eta^\circ > \Lambda$$

and the variables

$$\xi^* = \xi^\circ, \quad \eta^* = \eta^\circ \quad \text{for } \eta^\circ < \Lambda$$

$$\xi^* = \xi^\circ, \quad \eta^* = \left(1 - \frac{\rho_2}{\rho_1}\right)\Lambda + \frac{\rho_2}{\rho_1}\eta^\circ \quad \text{for } \eta^\circ > \Lambda$$

leaves the system (1.1) and the conditions (1.2) and (1.3) unchanged, with the quantities μ^0 and ν^0 replaced therein by the single parameter

$$k = \frac{\mu_2 \rho_2}{\mu_1 \rho_1}$$

Henceforth the index * on v_x, v_y, ξ, η will be omitted. The form of the dividing surface is easily determined for the case $k = 1$. Obviously on it

$$v_x = f'(\Lambda), \qquad v_y = \tfrac{1}{2}(\Lambda f' - f)$$

(where f is Blasius' solution), and condition (1.3) takes the form

$$\frac{f}{2}\frac{dw}{d\Lambda} - f'w + 1 = 0 \qquad (w = \xi^{-1})$$

Hence

$$w = -2f^2 \int_{C_1}^{\Lambda} f^{-3}\, d\Lambda$$

The constant C_1 is determined from the condition $w = 1$ at $\Lambda = \infty$. Therefore

$$w = -2f^2 \int_{\infty}^{\Lambda} f^{-3}\, d\Lambda$$

In the general case to solve the system (1.1) with boundary conditions (1.2), (1.3) we apply the method of integral relations. Integrating the momentum equation in (1.1) from zero to $\eta = \delta(\xi)$ (where $\delta(\xi)$ is the boundary layer thickness in the η, ξ variables), using the continuity equation and condition (1.3), we obtain the single integral relation

$$\xi \frac{d}{d\xi} \int_0^{\delta} v_x (v_x - 1)\, d\eta + \tfrac{1}{2} \int_0^{\delta} v_x (v_x - 1)\, d\eta - \xi^2 \frac{d}{d\xi} \int_0^{\delta} (v_x - 1)\, d\eta = -\left(\frac{\partial v_{x1}}{\partial \eta}\right)_0 \tag{1.4}$$

In the relation (1.4)

$$v_x = v_{x1} \quad \text{for } 0 \leqslant \eta \leqslant \Lambda, \qquad v_x = v_{x2} \quad \text{for } \Lambda \leqslant \eta \leqslant \delta$$

We describe the v_{x1} and v_{x2} velocity profiles by polynomials of the third degree:

$$v_{x1} = a_0 + a_1 \frac{\eta}{\delta} + a_2 \left(\frac{\eta}{\delta}\right)^2 + a_3 \left(\frac{\eta}{\delta}\right)^3,$$
$$v_{x2} = b_0 + b_1 \frac{\eta}{\delta} + b_2 \left(\frac{\eta}{\delta}\right)^2 + b_3 \left(\frac{\eta}{\delta}\right)^3$$

To determine the coefficients a_n and b_n we use, in addition to the conditions (1.2), also the following

$$\begin{aligned} &\frac{\partial^2 v_{x1}}{\partial \eta^2} = 0 \quad \text{at } \eta = 0, \qquad \frac{\partial v_{x2}}{\partial \eta} = 0 \quad \text{at } \eta = \delta(\xi) \\ &\frac{\partial^2 v_{x1}}{\partial \eta^2} = k \frac{\partial^2 v_{x2}}{\partial \eta^2}, \quad \frac{\partial^3 v_{x1}}{\partial \eta^3} = k^2 \frac{\partial^3 v_{x2}}{\partial \eta^3} \quad \text{at } \eta = \Lambda(\xi) \end{aligned} \tag{1.5}$$

which result from equations (1.1) and the definition of boundary layer thickness. As a result we obtain

$$v_{x1} = kb_3 \left\{ 3\,[(k-1)z^2 - 2(k-1)z - 1] \frac{\eta}{\delta} + k \left(\frac{\eta}{\delta}\right)^3 \right\} \tag{1.6}$$

$$v_{x2} = 1 + b_3 \left\{ 2 + 3(k-1)z - 3\,[2(k-1)z + 1] \frac{\eta}{\delta} + 3(k-1)z \left(\frac{\eta}{\delta}\right)^2 + \left(\frac{\eta}{\delta}\right)^3 \right\}$$

$$b_3 = -\tfrac{1}{2}\,[1 + 3(k-1)z + 3(k-1)^2 z^2 + (3k - 1 - 2k^2) z^3]^{-1} \qquad \left(z = \frac{\Lambda}{\delta}\right)$$

Substituting these expressions into relations (1.4) and (1.3) (where in the latter the value of v_y is determined from the continuity equation) we obtain a system of two ordinary first-order differential equations for the functions z and $y = \delta^2$:

$$
\begin{aligned}
&z f_0 y + 2\xi (f_0 + z f'_0 - \xi)\, y \frac{dz}{d\xi} + \xi z (f_0 - \xi) \frac{dy}{d\xi} = 0 \\
&2R - yP + 2\xi y (\xi Q' - P') \frac{dz}{d\xi} + \xi (\xi Q - P) \frac{dy}{d\xi} = 0
\end{aligned}
\tag{1.7}
$$

Here

$$
f_0(z) = \tfrac{1}{4} k\, [(7k-6) z^3 - 12 (k-1) z^2 - 6z]\, b_3(z)
$$

$$
R(z) = 3k\, [1 + 2(k-1) z + (1-k) z^2]\, b_3(z)
$$

$$
Q(z) = [\tfrac{3}{4} + (k-1) z + \tfrac{3}{2}(k-1) z^2 + 3(k-1)^2 z^3 + (-\tfrac{5}{4} + \tfrac{7}{2} k - \tfrac{9}{4} k^2) z^4]\, b_3(z)
$$

$$
\begin{aligned}
P(z) = [&-\tfrac{39}{70} + \tfrac{39}{10}(1-k) z - (\tfrac{87}{10} k^2 - \tfrac{102}{5} k + \tfrac{117}{10}) z^2 + (-6k^3 + 30k^2 - \tfrac{87}{2} k + \\
&+ \tfrac{39}{2}) z^3 + (7k^3 - \tfrac{71}{2} k^2 + 48k - \tfrac{39}{2}) z^4 + (-6k^4 + \tfrac{93}{10} k^3 + \tfrac{27}{2} k^2 - \tfrac{57}{2} k + \tfrac{117}{10}) z^5 + \\
&+ (k-1)(\tfrac{81}{10} k^3 - \tfrac{15}{2} k^2 - \tfrac{9}{2} k + \tfrac{39}{10}) z^6 + (-\tfrac{93}{35} k^4 + \tfrac{53}{10} k^3 - \tfrac{23}{10} k^2 - \tfrac{9}{10} k + \\
&+ \tfrac{39}{70}) z^7]\; b_3^2(z)
\end{aligned}
$$

Clearly the boundary conditions for (1.7) are

$$
z = 1, \quad y = \tfrac{280}{13} \qquad \text{at } \xi = 1
$$

Taking into account that

$$
z(0) = 0, \;\; y(0) = y_\circ = \tfrac{280}{13} \nu^\circ
$$

one can determine the behavior of the integral curves $z(\xi)$, $y(\xi)$ in the vicinity of the point $\xi = 0$.

In fact, assuming that

$$
z = \xi^n (a_1 + a_2 \xi + a_3 \xi^2 + a_4 \xi^3 + \ldots), \qquad y = y_0 [1 + \xi^m (b_1 + b_2 \xi + b_3 \xi^2 +)]
$$

and using (1.7) we obtain

$$
n = m = 1
$$

$$
\begin{aligned}
a_1 &= \frac{8}{15k}, & b_1 &= -\frac{16}{15}(k-1) \\
a_2 &= (k-1)\left(\frac{8}{15k}\right)^2, & b_2 &= \frac{64}{675} \frac{(k-1)(2k-3)}{k} \\
a_3 &= \frac{22k^2 - 25k + 6}{6}\left(\frac{8}{15k}\right)^3, & b_3 &= \frac{7}{5}\left(\frac{16}{117}\right)^2 \frac{(520 - 421k)(k-1)}{k} \\
a_4 &= \frac{(k-1)(13k^2 + 11k - 19)}{9}\left(\frac{8}{15k}\right)^4,
\end{aligned}
\tag{1.8}
$$

In the vicinity of the point $\xi = 1$ the integral curves $z(\xi)$ and $y(\xi)$ have the form:

$$
\begin{aligned}
\xi &= 1 + c_1(1-z) + c_2(1-z)^2 + c_3(1-z)^3 + c_4(1-z)^4 + c_5(1-z)^5 + \\
&\quad + c_6(1-z)^6 + \ldots \\
y(z) &= y_1\,[1 + d_1(1-z) + d_2(1-z)^2 + d_3(1-z)^3 + d_4(1-z)^4 + \\
&\quad + d_5(1-z)^5 + d_6(1-z)^6]
\end{aligned}
\tag{1.9}
$$

$c_1 = d_1 = 0$

$$
c_2 = \frac{\frac{32}{13}\,\frac{k-1}{k}}{\left(1 - \frac{280}{13y_1}\right) - \frac{110}{39}}, \qquad c_3 = \frac{8}{5}\,\frac{\frac{4-41k}{11k}\,\frac{k-1}{k} + 1}{\frac{39}{110}\left(1 - \frac{280}{13y_1}\right) - 1}
$$

$$
d_2 = \frac{k-1}{k}\,\frac{\frac{100}{13} - \frac{18}{5}\left(1 - \frac{280}{13y_1}\right)}{\frac{22}{13} - \frac{3}{5}\left(1 - \frac{280}{13y_1}\right)}
$$

$$
d_3 = \frac{80}{117k^2}\;\frac{30k^2 - 45k + 4}{\frac{110}{39} - \left(1 - \frac{280}{13y_1}\right)} - \frac{36k^2 - 54k + 10}{3k^2}
$$

$$
c_4 = -\frac{4}{39k^2} \times \frac{\frac{2}{13}\left(7846k^2 - 16132k + 8407\right) + [11 - 468(k-1)^2]\left(1 - \frac{280}{13y_1}\right)}{\left[\frac{110}{39} - \left(1 - \frac{280}{13y_1}\right)\right]^2}
$$

$$
\begin{aligned}
d_4 = \frac{153k^2 - 306k + 151}{3k^2} - \frac{\frac{1}{117k^2}}{\left[\frac{110}{39} - \left(1 - \frac{280}{13y_1}\right)^2\right]} &\left[\frac{435912k^2 - 2666560k + 1259080}{39} - \right. \\
&\left. - \left(12048k^2 - 24616k + 10996\right)\left(1 - \frac{280}{13y_1}\right)\right]
\end{aligned}
$$

Formulas (1.9) have been obtained for an arbitrary value of $y(1) = y_1$. The purpose of such generality will become clear later; for the case under consideration $y_1 = 280/13$.

We continue the integral curves for $z(\xi)$ and $y(\xi)$, the equations for which are given by formulas (1.8) and (1.9), to a value of the argument such that the two curves intersect or reach their minimum separation.

With a certain approximation it is possible to consider that the curves (continuous or discontinuous) obtained as the result of such continuation give the solution of the problem posed.

The accuracy of the assumptions made can be estimated from the following example. We set $k = 1$; then the solution of (1.7) satisfying the given boundary conditions has the form

$$y \equiv \tfrac{280}{13}, \qquad \frac{1}{\xi} = \frac{96\,(12+6z^2+7z^4) - 350z^6 + 35z^8}{2160\,z} + \frac{(z^4-6z^2)^2}{720} + \\ + \frac{7}{864\sqrt{6}}(z^4-6z^2)^2 \ln\frac{(\sqrt{6}-z)(\sqrt{6}+1)}{(\sqrt{6}+z)(\sqrt{6}-1)} \tag{1.10}$$

(We note that the corresponding analytical expression can also be obtained for $y_1 \neq 280/13$.)

For the case under consideration, formulas (1.8) and (1.9) give respectively

$$y \equiv \tfrac{280}{13}, \quad z = \tfrac{8}{15}\xi + \tfrac{1}{2}\left(\frac{8}{15}\xi\right)^3 + \ldots$$

$$y \equiv \tfrac{280}{13}, \quad z = 1 - (1-\xi)^{1/3}\left[b_0 - \frac{b_0^{-1}}{32}(1-\xi)^{1/3} - \tfrac{49}{3200}(1-\xi)^{2/3} + \frac{9533}{61440 b_0^2}(1-\xi)\right]$$

$$b_0 = \sqrt[3]{\tfrac{5}{8}}$$

In the table are shown values of the function $z(\xi)$ determined from formula (1.10) and from the proposed approximation. The discrepancy in the results does not exceed 2%.

TABLE

ξ	$z(\xi)$ from (1.10)	$z(\xi)$ from (1.8)	$z(\xi)$ from (1.9)	ξ	$z(\xi)$ from (1.10)	$z(\xi)$ from (1.8)	$z(\xi)$ from (1.9)
0	0	0		0.6	0.343	0.338	0.3336
0.1	0.052	0.052		0.65	0.376	0.369	0.369
0.2	0.106	0.106		0.7	0.410	0.3995	0.406
0.3	0.163	0.163		0.8	0.496		0.491
0.4	0.220	0.219		0.9	0.606		0.603
0.5	0.280	0.277		1	1		1

2. The results of Section 1 can be extended to the case of flow past a plate of a compressible fluid or gas for which the law of viscosity has the form $\mu_1\rho_1$ = const., $\mu_2\rho_2$ = const. Then, as follows from the work [2], the boundary layer equations in the variables

$$\xi^\circ = \xi = \frac{x}{V_\infty t}, \qquad \eta^\circ = \int_0^\eta \frac{\rho(\xi,\eta)}{\rho_{\infty 1}}\,d\eta \qquad \left(\eta = \frac{y}{\sqrt{x\nu_{\infty 1}/V_\infty}}\right)$$

for the functions

$$v^\circ_{xi} = v_{xi}, \quad v^\circ_{yi} = \xi(v_{xi} - \xi)\frac{\partial\eta^\circ}{\partial\xi} + \frac{\eta^\circ}{2}v_{xi} + \frac{\rho_i}{\rho_{\infty 1}}\sqrt{\frac{xV_\infty}{\nu_{\infty 1}}}\,\frac{v_{yi}}{V_\infty} - \frac{\rho_i}{\rho_{\infty 1}}\,\frac{v^\circ_{xi}\Lambda}{2}$$

have the same form as for an incompressible fluid, in which it is necessary to set $k = \mu_2\rho_2/\mu_{\infty 1}\rho_{\infty 1}$ (in the integration with respect to η and η°

the corresponding density profiles are used). The boundary conditions (1.2) and (1.3) also remain unchanged, which is confirmed by the following transformations:

$$v^\circ_{y1} = \xi (v_{x1})_\Lambda \left(\frac{\partial \eta^\circ}{\partial \xi}\right)_\Lambda - \xi^2 \left(\frac{\partial \eta^\circ}{\partial \xi}\right)_\Lambda + \left(\frac{\eta^\circ v_{x1}}{2}\right)_\Lambda + \left(\frac{\rho_1}{\rho_{\infty 1}}\right)_\Lambda \left[\frac{\Lambda}{2}(v_{x1})_\Lambda - \xi^2 \Lambda' + \right.$$

$$\left. + \xi \Lambda' (v_{x1})_\Lambda\right] - \frac{\Lambda}{2}(v_{x1})_\Lambda \left(\frac{\rho_1}{\rho_{\infty 1}}\right)_\Lambda = \xi (v_{x1})_\Lambda \frac{d\Lambda^\circ}{d\xi} - \xi^2 \frac{d\Lambda^\circ}{d\xi} + \left(\frac{v_{x1}\eta^\circ}{2}\right)_\Lambda = (v^\circ_{y2})_\Lambda$$

$$(\bar{v}^\circ_y)_\Lambda - \left(\frac{\eta^\circ v_x}{2}\right)_\Lambda = \xi^\circ [(v^\circ_x)_\Lambda - \xi^\circ] \frac{d\Lambda^\circ}{d\xi}$$

Consequently, for the case considered the solution obtained in Section 1 for the variables ξ^0, η^0 can be used for the velocity profile v_{xi}, the thickness of the dividing surface and the boundary layer thickness, if y_1 is determined in an appropriate manner. The latter is found from the condition of continuity at $\xi = 1$ of the boundary layer thickness in the ξ, η plane for a known value of the boundary layer thickness $y(1+)$ in the ξ^0, η^0 plane.

For the determination of the temperature profile it is necessary to find the solution of the energy equation

$$\xi^\circ (\dot{v}_{xi} - \xi^\circ) \frac{\partial \theta^\circ_i}{\partial \xi^\circ} + \left(v^\circ_{yi} - \frac{\eta^\circ}{2} v^\circ_{xi}\right) \frac{\partial \theta^\circ_i}{\partial \eta^\circ} = \frac{k_i}{P} \frac{\partial^2 \theta_i}{\partial \eta^{\circ 2}} + k_i \left(1 - \frac{1}{P}\right) \frac{\partial}{\partial \eta^\circ}\left(v^\circ_{xi} \frac{\partial \dot{v}_{xi}}{\partial \eta^\circ}\right)$$

$$\left(\theta_i = \frac{H_i}{V^2_\infty} + \frac{v^{\circ 2}_{xi}}{2}\right) \qquad (i = 1,2;\ k_1 = 1,\ k_2 = k) \qquad (2.1)$$

with the boundary conditions

$$H_1 = H_{\infty 1} \quad \text{for } \eta^\circ = \infty,\ \xi^\circ > 1; \qquad H_2 = H_{\infty 2} \quad \text{for } \eta^\circ = \infty, \quad \xi^\circ < 1$$

$$H_1 = H_w \quad \text{at } \eta^\circ = 0 \qquad (2.2)$$

$$T_1 = T_2, \qquad \frac{\partial H_1}{\partial \eta^\circ} = k \frac{P_1}{P_2} \frac{\partial H_2}{\partial \eta^\circ} \quad \text{at } \eta^\circ = \Lambda^\circ (\xi^\circ)$$

Here H_i is the enthalpy, P the Prandtl number. For the case $P_1 = P_2 = 1$, $c_{p1} = c_{p2}$ there exist the integrals

$$\theta^\circ_1 = \frac{H_w}{V^2_\infty} + \left(\theta^\circ_{02} - \frac{H_w}{V^2_\infty}\right) v^\circ_{x1}, \quad \theta^\circ_2 = \theta^\circ_{02} + \left(\theta^\circ_{02} - \frac{H_w}{V^2_\infty}\right)(v^\circ_{x2} - 1), \quad \xi < 1$$

$$\theta^\circ_1 = \frac{H_w}{V^2_\infty} + \left(\theta^\circ_{01} - \frac{H_w}{V^2_\infty} v^\circ_{x2}\right), \quad \xi > 1$$

where θ_{01} and θ_{02} denote the dimensionless stagnation enthalpies in the first and second gas. For $k = 1$, $p_1 = p_2$, $c_{p1} = c_{p2}$ there exist the usual integrals $\theta^0{}_i = \theta^0{}_i(v^0{}_{xi})$.

3. The results obtained in the preceding section can be used for the solution of the following problem. Let a shock wave run over a plate with velocity U, the velocity of the stream of gas behind the shock being V_∞.

Beginning at a certain moment it is replaced by a stream of a second gas. If the shock wave and the dividing surface of the gases appear simultaneously at the leading edge of the plate, then the problem under consideration is evidently self-similar. In this case formulas (1.8) and (1.9) can be used for its solution.

Here, as follows from [3], it is necessary to set

$$y(1+) = 8\left(\frac{U}{V_\infty} - 1\right)\left(\frac{U}{V_\infty} - \frac{13}{35}\right)^{-1}$$

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